PERFORMANCE EVALUATION OF YARD CRANES IN CONTAINER TERMINALS

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ABSTRACT

Various different types of yard cranes are used in container terminals. Examples are rubber tired gantry cranes, rail mounted gantry cranes, overhead bridge cranes, dual rail-mounted gantry cranes, and automated stacking cranes. The kinematics and handling characteristics of these yard cranes are different from each other. This study analyses characteristics of each type of yard crane and compares various performances - including throughput capacity and storage capacity - for different handling requirements.

Key Words: *Yard Cranes, Handling Characteristics, Performance.*

1. INTRODUCTION

Container terminal have three basic functions as follows: (a) delivering containers to outside trucks and receiving containers delivered by outside trucks, (b) loading containers onto and discharging containers from vessels, and (c) storing containers temporarily in the yard. At storage blocks in the yard, yard cranes carry out various container handling tasks which includes grounding - lifting containers from the trucks, internal trucks and external trucks, and storing them at storage locations - and picking-up - retrieving containers from their storage locations and putting them on the trucks.

It is important that appropriate handling equipment should be used according to type and operational characteristics of yard blocks for storing and retrieving containers. There are many different types of yard cranes which include rubber tired gantry cranes (RTGC), rail mounted gantry cranes (RMGC), overhead bridge cranes (OHBC), dual rail-mounted gantry cranes (DRMGC), and automated stacking cranes (ASC). Each type of yard cranes has its own unique characteristics. Furthermore, there are various types of layouts of blocks in the yard.

Until recently, researchers have paid little attention to the performance evaluation of yard cranes in container terminals. Kim and Kim (2002) and Wang (1998) have estimated several handling time models for dealing with deploying and sizing yard cranes. Castilho and Daganzo (1993) have proposed simple mathematical equations for the expectation and the variance of the number of moves required to retrieve a container from a yard bay. Hu *et al.* (2005) proposed expected travel time models for a new type of AS/RS which can be used in container terminals.

This study proposes detailed handling time models for yard cranes for various types of yard layout. These models may be used to estimate the performance of the yard cranes and selecting the most suitable type of yard cranes for a container terminal with specific operational requirements.

The rest of this paper is organized as follows: section 2 explains characteristics of each type of yard cranes and blocks. Section 3 provides formulas for expectation and variances of handling times of different types of yard cranes. In section 4, numeric examples are provided and performances of different type of cranes are compared with each other. Conclusions are given in section 5.

2. CHARACTERISTICS OF VARIOUS TYPES OF YARD CRANES AND BLOCKS

A rubber tired gantry crane (RTGC) is the most popular type of yard cranes in container terminals. It is operated manually, runs on rubber tired wheels, and can move from one block to another. RTGCs are used in terminals with blocks laid out parallel to the berth. Therefore, transfer points on which trucks wait for their transfer of containers by yard crane are located at side of each block.

A dual rail-mounted gantry crane (DRMGC) is being used at the Container Terminal Altenwerder (CTA) in Hamburg of Germany. DRMGC consists of a pair of yard cranes per block which are of different sizes and can cross each other. By installing two cranes of different sizes in the same block, it was possible to not only significantly increase the throughput rate of yard cranes in a block but also reduce interference between yard cranes. Also, it made container handling operation possible even when either of two yard cranes is broken down. Transfer between DRMGC and AGVs is carried out automatically. However, remote controllers in a control room are responsible to pick up (release) containers from (to) external trucks.

Automated stacking cranes (ASCs) are used at the European Combined of Terminal (ECT) in Rotterdam of Netherlands. Because the block length is somewhat shorter at the ECT than that in CTA, only one yard crane per each block is used. When an ASC is broken down, the ASC is towed out and a rescue crane is moved to the corresponding block to continue the handling operation.

Two rail mounted gantry cranes (RMGC) of the same size are used in each block at the Thames port in England. When one RMGC is broken down, the RMGC is towed out and the other one takes care of the entire range of the block. When picking-up (releasing) a container from (onto) external trucks, an operator with joy-stick control the first movement of cranes for picking and the final movement of cranes for landing a container.

An overhead bridge crane (OHBC) is being used for handling trans-shipment containers at the Pasir Panjang Terminal (PPT) in Singapore. It is the most expensive but the most productive yard crane. In a block with an OHBC, over-head rails and a travel lane for vehicles in the middle f the block are provided. Operators in a control room control picking-up and releasing operations.

RMGCs of the cantilever type are being used at the Hong Kong Internationl Terminal (HIT) in Hong Kong. RMGCs transfer containers from/to internal or external trucks parking at outside of legs of RMGCs. This type of RMGCs is usually used in terminals with blocks laid out in parallel direction to the berth.

Operation processes of yard cranes depend on the layout of blocks in the yard. A key characteristic of the yard, which is critical to determine the type of yard cranes, is locations of transfer points (TPs) on which vehicles park for receiving (delivering) containers from (to) yard cranes.

This study classifies types of yard blocks according to the location of transfer points and describes operational characteristics of each type of yard blocks. Furthermore, suitable combinations of the type of yard blocks and the type of yard cranes are suggested.

Figure 1 shows the first type of yard block which is the most popular in conventional container terminals. There are many TPs between trucks and yard cranes. An external truck (or internal truck) parks at a side of a bay where a container will be picked up from (put down onto) the truck. In this type of block layout, RTGCs, RMGCs of the cantilever, RMGCs of the rahmen, and OHBCs may be used as yard cranes.

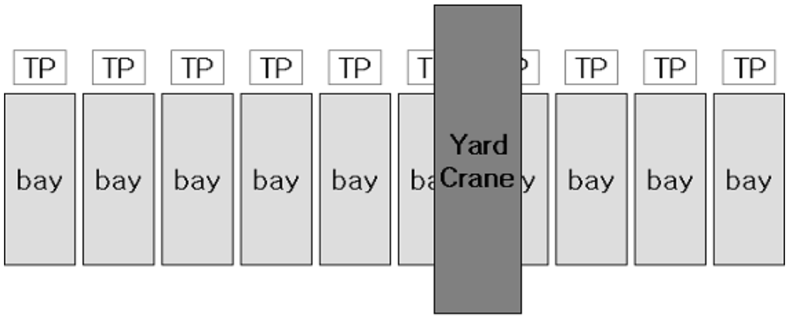

Figure 1. A block with TPs at a side of each bay

The second type of yard blocks is the same type of block as that in Figure 1, but, the number of TPs is less than the number of bays in a block. When an external truck (or internal truck) with a container arrives at a TP, a yard crane travels to that TP and picks up the container and travels to the storage location with the container. Examples of yard cranes, which are used in this type of blocks, are RMGCs of the rahmen type, RMGCs of the cantilever type, and OHBCs.

Figure 2 shows the most popular type of yard block in automated container terminals. There are several TPs at both ends of the block. The number of TPs ranges between 4 and 7 and TPs at both sides have different roles from each other. TPs of the water-side of a block are for internal vehicles and those of the land-side of a block are for external trucks. Thus, the traffic of external trucks is separated from that of internal vehicles which are fully automated.

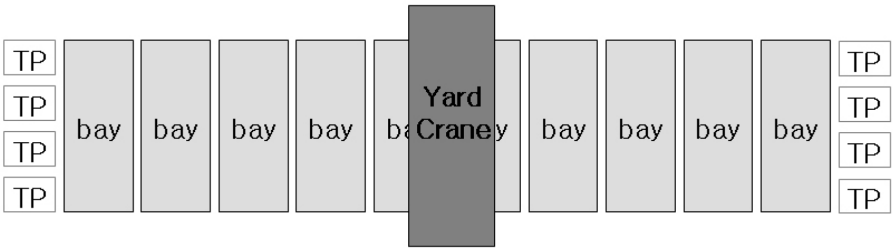

Figure 2. A block with TPs at the ends of a block

When an external truck (or internal vehicle) arrives at a TP, a yard crane travels to that TP and picks up a container. In order to store a container into a block, the yard crane moves back to a storage location with the container. Examples of yard cranes, which are used in this type of blocks, are DRMGCs and ASCs.

The next section addresses handling time models for yard cranes which are used in various types of yard block introduced above.

3. HANDLING TIME MODELS FOR YARD CRANES

The handling time depends on the type of handling equipment, its technical characteristics, and the degree of automation. A popular way to express the performance of a piece of handling equipment is to develop analytical models for the expectation and the variance of handling times.

This section proposes estimators for the expectation and the variance of cycle times, which can be used to estimate productivity of yard cranes for receiving, loading, unloading, and delivery operations. Following notations will be introduced for expressing cycle times.

3.1. Notations and basic expressions

Basic parameters for estimating cycle times are given in Table 1, Figure 3, and Figure 4.

Table 1. Notations

Notations	Descriptions
b	Number of bays
t	Number of tiers in a bay
r	Number of rows in a bay
m	Number of transfer points in either side of a block
l_0	Average number of consecutive loading at the same bay
c_w	Width of a container
c_h	Height of a container
c_l	Length of a 20ft container
d_c	Distance between at the end of bay and the centre of chassis location
d_b	Empty gap between two consecutive bays
d_r	Empty gap between two consecutive rows
h_c	Height of chassis
h_{max}	Height of the spreader at the top position
v_g^e	Speed of empty gantry travel of yard crane
v_g^l	Speed of loaded gantry travel of yard crane
v_t^e	Speed of empty trolley move of yard crane
v_t^l	Speed of loaded trolley move of yard crane
v_h^e	Speed of empty hoisting of yard crane
v_h^l	Speed of loaded hoisting of yard crane
D_t^h	Distance between the top position of the spreader and the target position of storing (retrieving) a container
d_{max}^h	Distance between the top position of the spreader and on the chassis, $d_{max}^h = h_{max} - (h_c + c_h)$
b_l	Block length, $b_l = (c_l + d_b)(b-1)$
b_w	Bay width, $b_w = (c_w + d_r)(r-1)$
s_{rt}	Storage capacity of a bay, $s_{rt} = r \times t - (r-1)$. $(r-1)$ is deducted by considering empty spaces necessary for relocations.
t_{max}^h	Hoisting time of the spreader for picking up a container
R_{rt}	Number of rehandles to pick up a random container from a bay with t tiers and r rows

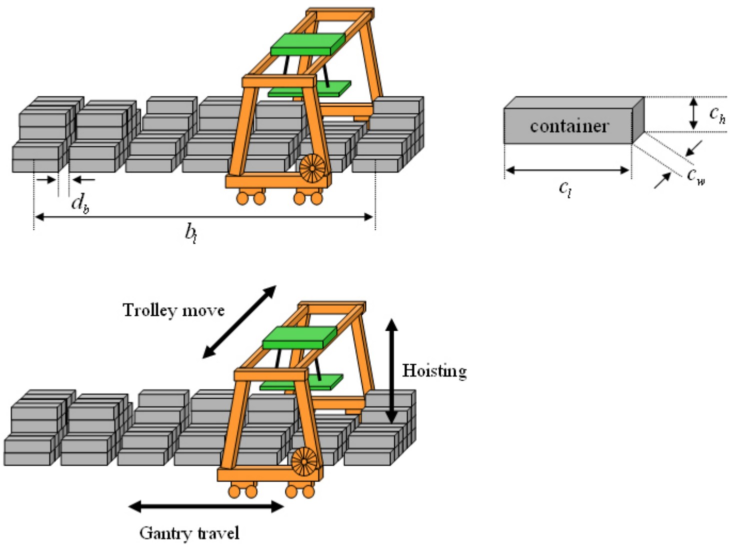

Figure 3. Illustrations of notations and terminologies on blocks

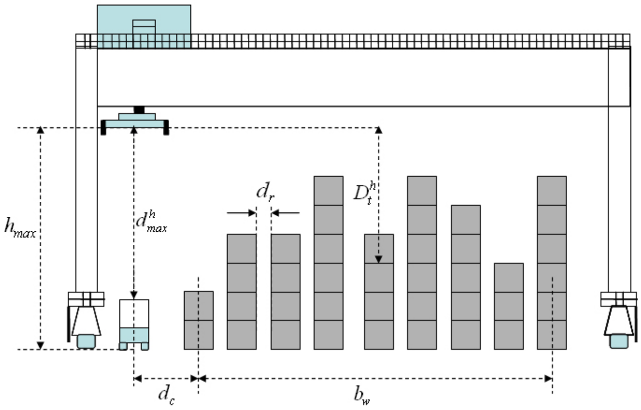

Figure 4. Illustrations of notations on a bay

The expected hoisting distance, $E\left(D_t^h\right)$, and the variance, $V\left(D_t^h\right)$, can be derived as follows:

$$E\left(D_t^h\right) = h_{max} - \frac{t+1}{2}c_h \text{, and} \tag{1}$$

$$V\left(D_t^h\right) = \frac{t^2-1}{12} \times c_h{}^2 . \tag{2}$$

The detailed description to get these expressions is given in Appendix A.

Under the assumption that retrievals of containers are performed in a random order, Kim (1997) proposed a formula to estimate the expected number of re-handle per pickup during all the containers are retrieved from a bay. He expressed it as a simple equation with the numbers of rows and tiers as follows:

$$E(R_{rt}) = \frac{t-1}{4} + \frac{t+2}{16r} . \tag{3}$$

The variance of number of rehandles for a random retrieval, $V(R_{rt})$, of a container from a bay was derived by using regression technique as follows:

$$V(R_{rt}) = -0.0186r + 0.0585t^2 + 0.2169 \,. \qquad (4)$$

3.2. The case that TPs are located at each side of bays (Block with a TP at each bay)

In this case, TPs are located at the side of each bay as shown in Figure 1. There are no physical structure for TPs, instead, the parking space at the side of each bay is used for trucks to receive (or deliver) containers from (to) yard cranes.

3.2.1. Expectation and variance of cycle times for the receiving operation

Let T_g be the gantry travel time of a yard crane for moving from a random location to another in a block. Then,

$$E\left(T_g\right) = \frac{b_l}{3} \times \frac{1}{v_g^e} \,. \qquad (5)$$

And, let t_{max}^h be the hoisting time a yard crane for picking up a container from a chassis and reaching to the chassis. Then,

$$t_{max}^h = d_{max}^h \left(\frac{1}{v_h^e} + \frac{1}{v_h^l} \right). \qquad (6)$$

Let T_t be the trolley travel time for the spreader to move in the trolley direction. Then,

$$E\left(T_t\right) = \left(\frac{b_w}{2} + d_c \right) \left(\frac{1}{v_t^e} + \frac{1}{v_t^l} \right). \qquad (7)$$

Finally, let T_h be the time for hoisting a container down on the stack and hoisting up the empty spreader. Then,

$$E\left(T_h\right) = E\left(D_t^h\right) \left(\frac{1}{v_h^e} + \frac{1}{v_h^l} \right). \qquad (8)$$

Thus, an expected cycle times per a receiving operation, $E\left(T_R\right)$, can be expressed as a summation of (5), (6), (7), (8). The time for grasping and releasing a container is excluded in the remaining part of this paper.

$E\left(T_R\right) = E\left(T_g\right) + t_{max}^h + E\left(T_t\right) + E\left(T_h\right)$. Thus,

$$E\left(T_R\right) = \frac{b_l}{3}\frac{1}{v_g^e} + \left(\frac{b_w}{2} + d_c \right) \left(\frac{1}{v_t^e} + \frac{1}{v_t^l} \right) + \left(d_{max}^h + E\left(D_t^h\right) \right) \left(\frac{1}{v_h^e} + \frac{1}{v_h^l} \right). \qquad (9)$$

In addition to the expectation, the variance of cycle times for the receiving operation, $V\left(T_R\right)$, can be expressed such as following (10).

$V\left(T_R\right) = V\left(T_g\right) + V\left(T_t\right) + V\left(T_h\right)$. Thus,

$$V\left(T_R\right) = \frac{b_l^2}{18} \left(\frac{1}{v_g^e} \right)^2 + \frac{b_w^2}{12} \left(\left(\frac{1}{v_t^e} \right)^2 + \left(\frac{1}{v_t^l} \right)^2 \right) + V\left(D_t^h\right) \left(\left(\frac{1}{v_h^e} \right)^2 + \left(\frac{1}{v_h^l} \right)^2 \right). \qquad (10)$$

3.2.2. Expectation and variance of cycle times for the loading operation

Because QCs load 5 - 20 containers of the same group consecutively and containers of the same group are usually stacked in the same bay, 5 - 20 loading operations are usually carried out by a yard crane at the same yard-bay. The expected cycle times for the loading operation, $E(T_L)$, can be derived in a similar way as that for the receiving operation as follows:

$$E(T_L) = E\left(\frac{T_g}{l_0}\right) + t_{max}^h + E(T_t) + E(T_h). \text{ Thus,}$$

$$E(T_L) = \frac{b_l}{3}\frac{1}{l_0}\frac{1}{v_g^e} + \left(\frac{b_w}{2} + d_c\right)\left(\frac{1}{v_t^e} + \frac{1}{v_t^l}\right) + \left(d_{max}^h + E\left(D_t^h\right)\right)\left(\frac{1}{v_h^e} + \frac{1}{v_h^l}\right). \quad (11)$$

Let N be the number of gantry travel between two consecutive loading operations. Then, the expectation and the variance are as follows:

$$E(N) = \frac{1}{l_0}, \text{ and} \quad (12)$$

$$V(N) = \left(0 - \frac{1}{l_0}\right)^2 \left(\frac{l_0 - 1}{l_0}\right) + \left(1 - \frac{1}{l_0}\right)^2 \frac{1}{l_0}$$

$$= \frac{l_0 - 1}{l_0^{\,2}}. \quad (13)$$

Therefore, the variance of the cycle times of the loading operations, $V(T_L)$, is expressed as follows:

$$V(T_L) = V\left(\sum_{i=1}^{N} T_g^i\right) + V(T_t) + V(T_h)$$

$$where,\ V\left(\sum_{i=1}^{N} T_g^i\right) = E(N)V(T_g) + E(T_g)^2 V(N). \text{ Thus,}$$

$$V(T_L) = \frac{1}{l_0}\left(\frac{3}{2} - \frac{1}{l_0}\right)\left(\frac{b_l}{3}\right)^2 \left(\frac{1}{v_g^e}\right)^2 + \frac{b_w^{\,2}}{12}\left(\left(\frac{1}{v_t^e}\right)^2 + \left(\frac{1}{v_t^l}\right)^2\right) + V\left(D_t^h\right)\left(\left(\frac{1}{v_h^e}\right)^2 + \left(\frac{1}{v_h^l}\right)^2\right). \quad (14)$$

Note that T_g^i is the loading time for the i^{th} loading operation. Equation (14) can be derived by the formula in the book by Ross (1996).

3.2.3. Expectation and variance of cycle times for the unloading operation

During the unloading operation, empty yard-bays are provided for unloading and the transfer of inbound containers is continued until all the slots in an empty yard-bay are fully filled with containers. Thus, the expected cycle times, $E(T_U)$, for the unloading operation can be estimated very similar with equation (11).

$$E(T_U) = E\left(\frac{T_g}{s_{rt}}\right) + t_{max}^h + E(T_t) + E(T_h). \text{ Thus,}$$

$$E(T_U) = \frac{b_l}{3}\frac{1}{s_{rt}}\frac{1}{v_g^e} + \left(\frac{b_w}{2} + d_c\right)\left(\frac{1}{v_t^e} + \frac{1}{v_t^l}\right) + \left(d_{max}^h + E\left(D_t^h\right)\right)\left(\frac{1}{v_h^e} + \frac{1}{v_h^l}\right). \quad (15)$$

And, the variance of cycle times, $V(T_U)$, for the unloading operation can be derived in the same way in section 3.2.2. In this case, N is defined the number of gantry travel between two consecutive unloading operations, which is s_{rt}.

$$V(T_U) = V\left(\sum_{i=1}^{N} T_g^i\right) + V(T_t) + V(T_h)$$

$$where,\ V\left(\sum_{i=1}^{N} T_g^i\right) = E(N)V(T_g) + E(T_g)^2 V(N).\ \text{Thus,}$$

$$V(T_U) = \frac{1}{s_{rt}}\left(\frac{3}{2} - \frac{1}{s_{rt}}\right)\left(\frac{b_l}{3}\right)^2\left(\frac{1}{v_g^e}\right)^2 + \frac{b_w^{\ 2}}{12}\left(\left(\frac{1}{v_t^e}\right)^2 + \left(\frac{1}{v_t^l}\right)^2\right) + V\left(D_t^h\right)\left(\left(\frac{1}{v_h^e}\right)^2 + \left(\frac{1}{v_h^l}\right)^2\right). \qquad (16)$$

3.2.4. Expectation and variance of cycle times for the delivery operation

The cycle times for the delivery operation consists of the handling time for a target container, the travel time between yard-bay, and the rehandling time. In order to estimate the rehandling time for the delivery operation, the number of rehandles in a bay should be derived as a simple expression. The rehandling work influences the performance of yard cranes significantly. Therefore, rehandling time must be considered for estimating cycle times.

In order to consider the number of relocations, The expected value and the variance of the cycle times for the delivery operation can be expressed by using equation (3), (5), (6), (7), and (8). The expected cycle times for the delivery operation consists of an $E(T_g)$, a t_{max}^h, an $E(T_t)$, $E(R_{rt})$ times of $\frac{b_w}{3}\left(\frac{1}{v_t^e} + \frac{1}{v_t^l}\right)$, and $\left(2E(R_{rt})+1\right)$ times of $E(T_h)$.

Thus, the expected cycle times for the delivery operation can be derived by using equation (3) as follows:

$$E(T_D) = E(T_g) + t_{max}^h + E(T_t) + \frac{b_w}{3}\left(\frac{1}{v_t^e} + \frac{1}{v_t^l}\right)E(R_{rt}) + \left(2E(R_{rt})+1\right)E(T_h).\ \text{Thus,}$$

$$E(T_D) = \frac{b_l}{3}\frac{1}{v_g^e} + \left(\left(\frac{1}{2} + \frac{E(R_{rt})}{3}\right)b_w + d_c\right)\left(\frac{1}{v_t^e} + \frac{1}{v_t^l}\right) + \left(d_{max}^h + \left(2E(R_{rt})+1\right)E\left(D_t^h\right)\right)\left(\frac{1}{v_h^e} + \frac{1}{v_h^l}\right). \qquad (17)$$

The trolley travel for relocations is repeated $E(R_{rt})$ times. Also, the hoisting operation for the relocations is repeated $\left(2E(R_{rt})+1\right)$ times. Let T_{rel}^i be a trolley time for i^{th} relocation. Then, $E\left(T_{rel}^i\right) = \frac{b_w}{3}\left(\frac{1}{v_t^e} + \frac{1}{v_t^l}\right)$, and the variance, $V\left(T_{rel}^i\right) = \frac{b_w^{\ 2}}{18}\left(\left(\frac{1}{v_t^e}\right)^2 + \left(\frac{1}{v_t^l}\right)^2\right)$. Let T_h^i and $T_{h'}^i$ be the i^{th} hoisting time at the origin stack and the destination stack of the relocated container, respectively. Then, the variance of cycle times of the delivery operation can be obtained as follows:

$$V(T_D) = V(T_g) + V(T_t) + V\left(\sum_{i=1}^{R_{rt}} T_{rel}^i\right) + V\left(\sum_{i=1}^{R_{rt}} \left(T_h^i + T_{h'}^i\right)\right) + V(T_h)$$

$$where,\ V\left(\sum_{i=1}^{R_{rt}} T_{rel}^i\right) = E(R_{rt})V(T_{rel}) + E(T_{rel})^2 V(R_{rt})$$

$$and,\ V\left(\sum_{i=1}^{R_{rt}} \left(T_h^i + T_{h'}^i\right)\right) = 2E(R_{rt})V(T_h) + 4E(T_h)^2 V(R_{rt}).\ \text{Thus,}$$

$$V(T_D) = \frac{b_l^{\,2}}{18}\left(\frac{1}{v_g^e}\right)^2 + b_w^{\,2}\left(\frac{1}{12} + \frac{E(R_{rt})}{18}\right)\left(\left(\frac{1}{v_t^e}\right)^2 + \left(\frac{1}{v_t^l}\right)^2\right) + V(R_{rt})\frac{b_w^{\,2}}{9}\left(\frac{1}{v_t^e} + \frac{1}{v_t^l}\right)^2$$
$$+\left(2E(R_{rt})+1\right)V\left(D_t^h\right)\left(\left(\frac{1}{v_h^e}\right)^2 + \left(\frac{1}{v_h^l}\right)^2\right) + 4V(R_{rt})E\left(D_t^h\right)^2\left(\frac{1}{v_h^e} + \frac{1}{v_h^l}\right)^2 . \quad (18)$$

3.3. The case that TPs are located intermittently at the side of the block (Block with intermittent TPs)

In this type of blocks, trucks are parked at one of physical TPs at the side of block, and a yard crane has to travel with a container between a TP and a storage position within its operation range in the block. The operation range is determined by the number of TPs. Therefore, trucks must park a specific TP to store (retrieve) their containers.

3.3.1. Expectation and variance of cycle times for the receiving operation

Expectation and variance of cycle times of the receiving operations can be calculated as follows: Because the number of TPs is less than the number of yard-bays, each TP has its operation range which covers more than one bay. And each transfer point is located at the middle of its range. The expected travel time from the position of a yard crane, which is a random position within a block, to a random TP can be expressed as the following. Note that positions of TPs can be represented as $b_l(2i-1)/2m$ *for* $i = 1,2,...,m$. Then,

$$E(T_g) = \frac{b_l}{m}\sum_{i=1}^{m}\left(\int_0^1 \left|x - \frac{2i-1}{2m}\right| dx\right)\frac{1}{v_g^e} . \quad (19)$$

Furthermore, the expected travel time from a TP to a random storage bay within the range allocated to the TP can be expressed as follows:

$$E\left(T_g^{re}\right) = \frac{1}{2}\left(\frac{b_l}{2m}\right)\frac{1}{v_g^e},\ \text{and} \quad (20)$$

$$E\left(T_g^{rl}\right) = \frac{1}{2}\left(\frac{b_l}{2m}\right)\frac{1}{v_g^l} . \quad (21)$$

The other equations are the same as equations in section 3.2.1. Therefore, the expected cycle time for the receiving operation is expressed as follows.

$$E(T_R) = E(T_g) + t_{max}^h + E\left(T_g^{rl}\right) + E(T_t) + E(T_h).\ \text{Thus,}$$

$$E(T_R) = \frac{b_l}{m}\left\{\frac{1}{v_g^e}\sum_{i=1}^{m}\left(\int_0^1 \left|x - \frac{2i-1}{2m}\right| dx\right) + \frac{1}{4v_g^l}\right\}$$

$$+\left(\frac{b_w}{2}+d_c\right)\left(\frac{1}{v_t^e}+\frac{1}{v_t^l}\right)+\left(d_{max}^h+E\left(D_t^h\right)\right)\left(\frac{1}{v_h^e}+\frac{1}{v_h^l}\right). \tag{22}$$

The variance of cycle times for the receiving operation can be derived as follows. First, we will derive the expression for the variance of travel distance of a yard crane in gantry direction from any position in a block to specific TP.

$$V\left(D_g\right)=\frac{b_l^{\,2}}{m}\sum_{i=1}^{m}\left(\int_0^1\left|x-\frac{2i-1}{2m}\right|dx\right)^2-\left(\frac{b_l}{m}\sum_{i=1}^{m}\left(\int_0^1\left|x-\frac{2i-1}{2m}\right|dx\right)\right)^2. \tag{23}$$

Therefore, the variance of cycle times for the receiving operation is expressed by

$V\left(T_R\right)=V\left(T_g\right)+V\left(T_g^{rl}\right)+V\left(T_t\right)+V\left(T_h\right)$, Thus

$$V\left(T_R\right)=V\left(D_g\right)\left(\frac{1}{v_g^e}\right)^2+\frac{1}{12}\left(\frac{b_l}{2m}\right)^2\left(\frac{1}{v_g^l}\right)^2$$

$$+\frac{b_w^{\,2}}{12}\left(\left(\frac{1}{v_t^e}\right)^2+\left(\frac{1}{v_t^l}\right)^2\right)+V\left(D_t^h\right)\left(\left(\frac{1}{v_h^e}\right)^2+\left(\frac{1}{v_h^l}\right)^2\right). \tag{24}$$

3.3.2. Expectation and variance of cycle times for the loading operation

The expected cycle times and the variance of cycle times for the loading operation can be derived in the same way in section 3.2.2. Some points of different can be expressed by equation (20) and (21). As the expected gantry travel time, equation (5) $E\left(T_g\right)$, is used.

$E\left(T_L\right)=E\left(\frac{T_g}{l_0}\right)+E\left(\frac{l_0-1}{l_0}T_g^{re}\right)+E\left(T_g^{rl}\right)+t_{max}^h+E\left(T_t\right)+E\left(T_h\right)$, Thus

$$E\left(T_L\right)=\left(\frac{b_l}{3}\frac{1}{l_0}+\frac{1}{2}\left(\frac{l_0-1}{l_0}\right)\left(\frac{b_l}{2m}\right)\right)\frac{1}{v_g^e}+\frac{1}{2}\left(\frac{b_l}{2m}\right)\frac{1}{v_g^l}+\left(\frac{b_w}{2}+d_c\right)\left(\frac{1}{v_t^e}+\frac{1}{v_t^l}\right)$$

$$+\left(d_{max}^h+E\left(D_t^h\right)\right)\left(\frac{1}{v_h^e}+\frac{1}{v_h^l}\right). \tag{25}$$

Let N_1 correspond to N in section 3.2.2, and let N_2 be defined to be the number of empty travel in gantry direction within the allocated range between two consecutive loading operations. It was founded that variances of N_1 and N_2 are same. By a simple derivation, we can have the variance of cycle times for the loading operation as follow:

$$V\left(T_L\right)=V\left(\sum_{i=1}^{N_1}T_g^i\right)+V\left(\sum_{i=1}^{N_2}T_g^{re(i)}\right)+V\left(T_g^{rl}\right)+V\left(T_t\right)+V\left(T_h\right)$$

$$where,\ V\left(\sum_{i=1}^{N_1}T_g^i\right)=E\left(N_1\right)V\left(T_g\right)+E\left(T_g\right)^2V\left(N_1\right)$$

$$and\ V\left(\sum_{i=1}^{N_2}T_g^{re(i)}\right)=E\left(N_2\right)V\left(T_g^{re}\right)+E\left(T_g^{re}\right)^2V\left(N_2\right).$$ Thus,

$$V\left(T_L\right)=\left(\frac{1}{l_0}\left(\frac{3}{2}-\frac{1}{l_0}\right)\left(\frac{b_l}{3}\right)^2+\frac{1}{4}\left(\frac{l_0-1}{l_0}\right)\left(\frac{1}{3}+\frac{1}{l_0}\right)\left(\frac{b_l}{2m}\right)^2\right)\left(\frac{1}{v_g^e}\right)^2$$

$$+\frac{1}{12}\left(\frac{b_l}{2m}\right)^2\left(\frac{1}{v_g^l}\right)^2+\frac{b_w^{\ 2}}{12}\left(\left(\frac{1}{v_t^e}\right)^2+\left(\frac{1}{v_t^l}\right)^2\right)+V\left(D_t^h\right)\left(\left(\frac{1}{v_h^e}\right)^2+\left(\frac{1}{v_h^l}\right)^2\right). \qquad (26)$$

3.3.3. Expectation and variance of cycle times for the unloading operation

The expected cycle times and the variance of cycle times for the unloading operation are almost similar to equation (25) and (26). The parameter l_0 is replaced by s_{rt}, and as the expected gantry travel time, $E\left(T_g\right)$ equation (19) is used. Thus, the expected cycle times, $E\left(T_U\right)$, and the variance, $V\left(T_U\right)$, can be derived as follows:

$$E\left(T_U\right)=E\left(\frac{T_g}{s_{rt}}\right)+E\left(\frac{s_{rt}-1}{s_{rt}}T_g^{re}\right)+E\left(T_g^{rl}\right)+t_{max}^h+E\left(T_t\right)+E\left(T_h\right). \text{ Thus,}$$

$$E\left(T_U\right)=\frac{b_l}{m}\frac{1}{s_{rt}}\left[\frac{1}{v_g^e}\sum_{i=1}^{m}\left(\int_0^1\left|x-\frac{2i-1}{2m}\right|dx\right)+\frac{\left(2s_{rt}-1\right)}{4v_g^l}\right]$$
$$+\left(\frac{b_w}{2}+d_c\right)\left(\frac{1}{v_t^e}+\frac{1}{v_t^l}\right)+\left(d_{max}^h+E\left(D_t^h\right)\right)\left(\frac{1}{v_h^e}+\frac{1}{v_h^l}\right). \qquad (27)$$

Let N_1 correspond to N in section 3.2.3, and let N_2 be the number of empty travel in gantry direction within the allocated range between two consecutive unloading operations. Then,

$$V\left(T_U\right)=V\left(\sum_{i=1}^{N_1}T_g^i\right)+V\left(\sum_{i=1}^{N_2}T_g^{re(i)}\right)+V\left(T_g^{rl}\right)+V\left(T_t\right)+V\left(T_h\right). \text{ Thus,}$$

$$V\left(T_U\right)=\frac{1}{s_{rt}}\left(V\left(D_g\right)+\left(\frac{b_l}{m}\sum_{i=1}^{m}\left(\int_0^1\left|x-\frac{2i-1}{2m}\right|dx\right)\right)^2\left(\frac{s_{rt}-1}{s_{rt}}\right)+\frac{\left(s_{rt}-1\right)\left(s_{rt}+3\right)}{12s_{rt}}\left(\frac{b_l}{2m}\right)^2\right)\left(\frac{1}{v_g^e}\right)^2$$
$$+\frac{1}{12}\left(\frac{b_l}{2m}\right)^2\left(\frac{1}{v_g^l}\right)^2+\frac{b_w^{\ 2}}{12}\left(\left(\frac{1}{v_t^e}\right)^2+\left(\frac{1}{v_t^l}\right)^2\right)+V\left(D_t^h\right)\left(\left(\frac{1}{v_h^e}\right)^2+\left(\frac{1}{v_h^l}\right)^2\right). \qquad (28)$$

3.3.4. Expectation and variance of cycle times for the delivery operation

In this type of block, the expected value and the variance of the cycle time for the delivery operation can be classified by using equation (3), (5), (21), (6), (7), and (8). The expected cycle times for the delivery operation consists of an $E\left(T_g\right)$, a t_{max}^h, an $E\left(T_g^{rl}\right)$, an $E\left(T_t\right)$, $E(R_{rt})$ times of $\frac{b_w}{3}\left(\frac{1}{v_t^e}+\frac{1}{v_t^l}\right)$, and $\left(2E(R_{rt})+1\right)$ times of $E\left(T_h\right)$.

Thus, the expected cycle times and the variance of cycle times for the delivery operation can be derived by considering equation (3) and (4), respectively, as follows:.

$$E\left(T_D\right)=E\left(T_g\right)+t_{max}^h+E\left(T_g^{rl}\right)+E\left(T_t\right)+\frac{b_w}{3}\left(\frac{1}{v_t^e}+\frac{1}{v_t^l}\right)E\left(R_{rt}\right)+\left(2E\left(R_{rt}\right)+1\right)E\left(T_h\right). \text{ Thus,}$$

$$E\left(T_D\right)=\frac{b_l}{3}\frac{1}{v_g^e}+\frac{1}{2}\left(\frac{b_l}{2m}\right)\frac{1}{v_g^l}+\left(\left(\frac{1}{2}+\frac{E\left(R_{rt}\right)}{3}\right)b_w+d_c\right)\left(\frac{1}{v_t^e}+\frac{1}{v_t^l}\right)$$

$$+\left(d_{max}^{h}+\left(2E\left(R_{rt}\right)+1\right)E\left(D_{t}^{h}\right)\right)\left(\frac{1}{v_{h}^{e}}+\frac{1}{v_{h}^{l}}\right), \text{ and} \quad (29)$$

$$V\left(T_{D}\right)=V\left(T_{g}\right)+V\left(T_{g}^{rl}\right)+V\left(T_{t}\right)+V\left(\sum_{i=1}^{R_{rt}}T_{tmp}^{i}\right)+V\left(\sum_{i=1}^{R_{rt}}\left(T_{h}^{i}+T_{h'}^{i}\right)\right)+V\left(T_{h}\right). \text{ Thus,}$$

$$V\left(T_{D}\right)=\frac{b_{l}^{2}}{18}\left(\frac{1}{v_{g}^{e}}\right)^{2}+\frac{1}{12}\left(\frac{b_{l}}{2m}\right)^{2}\left(\frac{1}{v_{g}^{l}}\right)^{2}$$
$$+b_{w}^{2}\left(\frac{1}{12}+\frac{E\left(R_{rt}\right)}{18}\right)\left(\left(\frac{1}{v_{t}^{e}}\right)^{2}+\left(\frac{1}{v_{t}^{l}}\right)^{2}\right)+V\left(R_{rt}\right)\frac{b_{w}^{2}}{9}\left(\frac{1}{v_{t}^{e}}+\frac{1}{v_{t}^{l}}\right)^{2}$$
$$+\left(2E\left(R_{rt}\right)+1\right)V\left(D_{t}^{h}\right)\left(\left(\frac{1}{v_{h}^{e}}\right)^{2}+\left(\frac{1}{v_{h}^{l}}\right)^{2}\right)+4V\left(R_{rt}\right)E\left(D_{t}^{h}\right)^{2}\left(\frac{1}{v_{h}^{e}}+\frac{1}{v_{h}^{l}}\right)^{2}. \quad (30)$$

3.4. The case that TPs are located at both ends of the blocks (Block with TPs at ends)

In this case, blocks are laid out perpendicular to the wharf. Sea-side and land-side traffic are separated. Thus, the traffic flow of trucks is simple compare with the case with blocks parallel to the wharf.

3.4.1. Expectation and variance of cycle times for the receiving operation

In this model, the expected gantry travel distance between moves follows uniform distribution, $U\left(0,\, b_{l}\right)$. The yard crane moves from one position to any randomized position in a block. Therefore, the expected round-trip gantry travel time becomes

$$E\left(T_{g}\right)=\frac{b_{l}}{2}\left(\frac{1}{v_{g}^{e}}+\frac{1}{v_{g}^{l}}\right). \quad (31)$$

Trolley travel time of the spreader can be written as

$$E\left(T_{t}\right)=\left(\frac{b_{w}}{3}+d_{c}\right)\left(\frac{1}{v_{t}^{e}}+\frac{1}{v_{t}^{l}}\right). \quad (32)$$

The expected cycle times for the receiving operation can be expressed by adding several basic elementary travel times.

$$E\left(T_{R}\right)=E\left(T_{g}\right)+t_{max}^{h}+E\left(T_{t}\right)+E\left(T_{h}\right). \text{ Thus,}$$

$$E\left(T_{R}\right)=\frac{b_{l}}{2}\left(\frac{1}{v_{g}^{e}}+\frac{1}{v_{g}^{l}}\right)+\left(\frac{b_{w}}{3}+d_{c}\right)\left(\frac{1}{v_{t}^{e}}+\frac{1}{v_{t}^{l}}\right)+\left(d_{max}^{h}+E\left(D_{t}^{h}\right)\right)\left(\frac{1}{v_{h}^{e}}+\frac{1}{v_{h}^{l}}\right). \quad (33)$$

The variance of cycle times for the receiving operation can be expressed as equation (34).

$$V\left(T_{R}\right)=V\left(T_{g}\right)+V\left(T_{t}\right)+V\left(T_{h}\right). \text{ Thus,}$$

$$V\left(T_{R}\right)=\frac{b_{l}^{2}}{12}\left(\left(\frac{1}{v_{g}^{e}}\right)^{2}+\left(\frac{1}{v_{g}^{l}}\right)^{2}\right)+\frac{b_{w}^{2}}{18}\left(\left(\frac{1}{v_{t}^{e}}\right)^{2}+\left(\frac{1}{v_{t}^{l}}\right)^{2}\right)+V\left(D_{t}^{h}\right)\left(\left(\frac{1}{v_{h}^{e}}\right)^{2}+\left(\frac{1}{v_{h}^{l}}\right)^{2}\right). \quad (34)$$

3.4.2. Expectation and variance of cycle times for the loading operation

The expectation, $E(T_L)$, and the variance, $V(T_L)$, of cycle times for the loading operation are the same as $E(T_R)$ of equation (33) and $V(T_R)$ of equation (34), respectively.

3.4.3. Expectation and variance of cycle times for the unloading operation

The expectation, $E(T_U)$, and the variance, $V(T_U)$, of cycle times for the unloading operation are the same as, $E(T_R)$ of equation (33) and $V(T_R)$ of equation (34), respectively.

3.4.4. Expectation and variance of cycle times for the delivery operation

The expectation and the variance of cycle times for the delivery operation can be expressed by using equation (3), (31), (6), (32), and (8). The expected cycle times for the delivery operation consists of an $E(T_g)$, a t_{max}^h, an $E(T_t)$, $E(R_{rt})$ times of $\frac{b_w}{3}\left(\frac{1}{v_t^e}+\frac{1}{v_t^l}\right)$, and $\left(2E(R_{rt})+1\right)$ times of $E(T_h)$.

The expectation and the variance of cycle times for the delivery operation can be derived by using equation (3) and (4) as follows.

$$E(T_D)=E(T_g)+t_{max}^h+E(T_t)+\frac{b_w}{3}\left(\frac{1}{v_t^e}+\frac{1}{v_t^l}\right)E(R_{rt})+\left(2E(R_{rt})+1\right)E(T_h). \text{ Thus,}$$

$$E(T_D)=\frac{b_l}{2}\left(\frac{1}{v_g^e}+\frac{1}{v_g^l}\right)+\left(\left(1+E(R_{rt})\right)\frac{b_w}{3}+d_c\right)\left(\frac{1}{v_t^e}+\frac{1}{v_t^l}\right)$$
$$+\left(d_{max}^h+\left(2E(R_{rt})+1\right)E\left(D_t^h\right)\right)\left(\frac{1}{v_h^e}+\frac{1}{v_h^l}\right), \text{ and} \quad (35)$$

$$V(T_D)=V(T_g)+V(T_t)+V\left(\sum_{i=1}^{R_{rt}}T_{rel}^i\right)+V\left(\sum_{i=1}^{R_{rt}}\left(T_h^i+T_{h'}^i\right)\right)+V(T_h). \text{ Thus,}$$

$$V(T_D)=\frac{b_l^2}{12}\left(\left(\frac{1}{v_g^e}\right)^2+\left(\frac{1}{v_g^l}\right)^2\right)+\frac{b_w^2}{18}\left(E(R_{rt})+1\right)\left(\left(\frac{1}{v_t^e}\right)^2+\left(\frac{1}{v_t^l}\right)^2\right)+\frac{b_w^2}{9}V(R_{rt})\left(\frac{1}{v_t^e}+\frac{1}{v_t^l}\right)^2$$
$$+\left(2E(R_{rt})+1\right)V\left(D_t^h\right)\left(\left(\frac{1}{v_h^e}\right)^2+\left(\frac{1}{v_h^l}\right)^2\right)+4V(R_{rt})E\left(D_t^h\right)^2\left(\frac{1}{v_h^e}+\frac{1}{v_h^l}\right)^2. \quad (36)$$

4. NUMERIC EXAMPLES

A numerical experiment was conducted for comparing performances of various types of blocks. In order to evaluate the performance, it was assumed that t=4, r = 6, b = 30, d_r = d_b = 0.4, $h_{\max}$ = 21 m, h_c = 1.5 m, d_c = 6 m, v_g^e = v_g^l = 150 m/min, v_t^e = v_t^l = 120 m/min, v_h^e = v_h^l = 75 m/min, and m = 6. It was assumed that only 20-ft containers are stacked in the yard, that is, c_l =6.058 m, c_w =2.438 m, and c_h =2.438 m.

This section calculates and compares some cycle times for the delivery operation of various yard blocks. Figure 5 shows expected cycle times of different types of yard blocks for the delivery operation. The number of TPs was set to 6 in case of blocks with intermittent TPs.

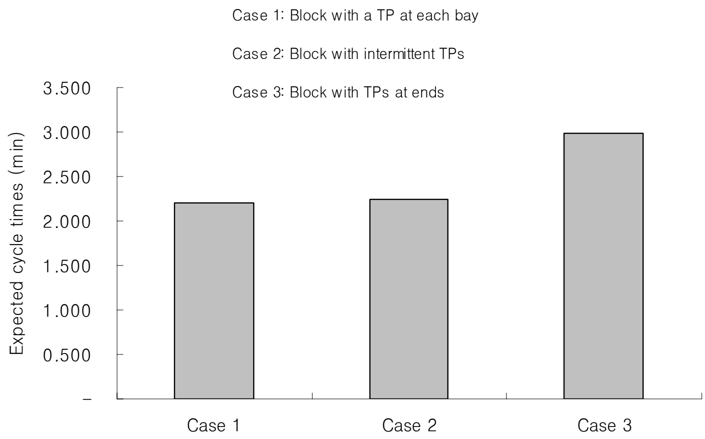

Figure 5. Expected cycle times of various types of blocks for the delivery operation

Expectations of cycle times in blocks with TPs at the side of the block were lower than that in blocks with TPs at the ends of the block. It comes from additional gantry travel time between pickups. Furthermore, variances of cycle times are compared in Figure 6.

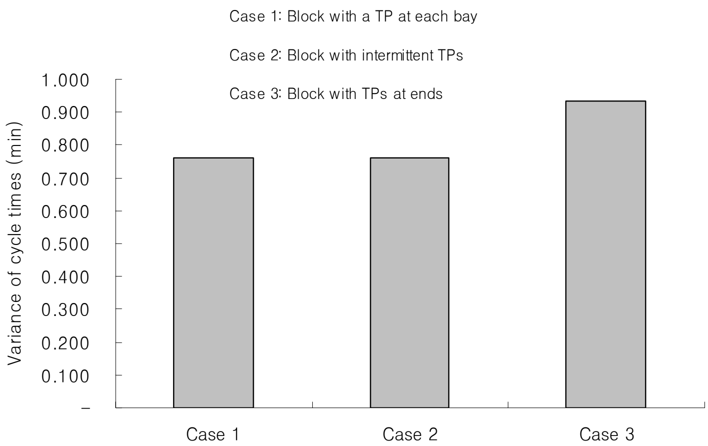

Figure 6. Variances of cycle times of various types of blocks for the delivery operation

Variances of cycle times in blocks with TPs at the side of the block were also much lower than that in blocks with TPs at the ends of the block. This difference also comes from the longer gantry travel time of yard cranes in blocks with TPs at ends than that in blocks with TPs at side.

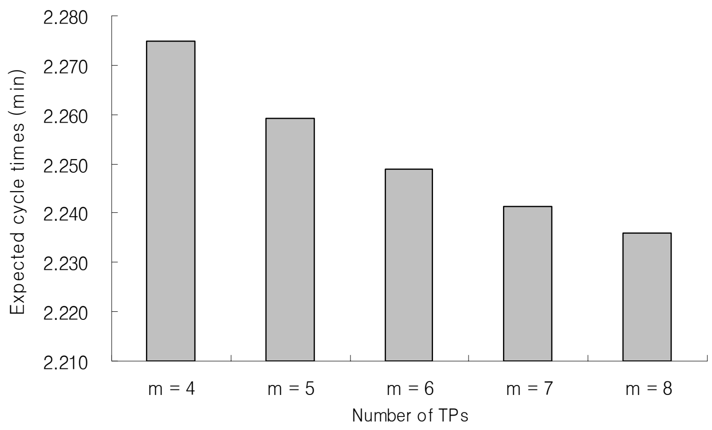

Figure 7. Expected cycle times of blocks with intermittent TPs during the delivery operation for different number of TP

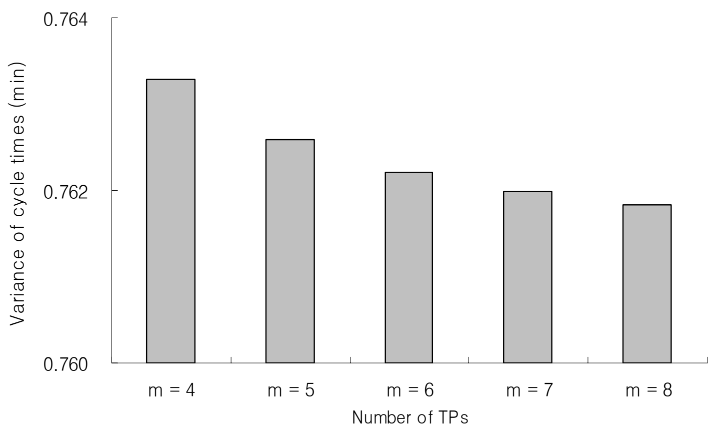

Figure 8. Variances of cycle times in blocks with intermittent TPs for the delivery operation

For the case of blocks with intermittent TPs, Figure 7 shows the changes of the expected cycle times for different number of TPs. Figure 7 shows that, as the number of TPs increases, the expected travel distance of yard cranes become shorter. As a result, the expected cycle time decreases. The variance of cycle time also decreases as the number of TPs increases, as shown in Figure 8.

5. CONCLUSION

This study proposed formulas for estimating the cycle time of various types of yard cranes in container terminals. Handling times of a yard crane consists of the travel time between yard-bays, the hoisting time of the spreader, and the trolley travel time of the spreader.

This study addressed three different types of yard blocks. The first one is the block whose a TP is located at the side of each bay in the block. The second is the block whose TPs are located at ends of the block. Yard cranes must do a round-trip travel for a storage/retrieval operation in this case. The last is the block whose TPs are located at several selected positions at the side of the block. The number of TP is a critical factor to determine the performance of the yard crane.

Furthermore, expectations and variances of the cycle time for the receiving, loading, unloading, and delivery operations are formulated analytically. These can be used easily for estimating the performance of yard cranes and yard blocks. It can be used to design or evaluate a new type of yard cranes.

All mathematical models in this paper must be proved by a simulation test. It is one of future promising studies.

ACKNOWLEDGEMENTS

This paper is a part of the result of the project "Development of Intelligent Port and Logistics System for Super-Large Container Ships", which was sponsored by Ministry of Maritime and Fishery in Korea.

APPENDIX A. ESTIMATION OF EXPECTED HOISTING DISTANCE AND ITS VARIANCE

We assume that the hoisting distance is continuous. Therefore, we can estimate the expected hosting distance from the maximum spreader height to the stacking position of a stack by using following formula.

$$\begin{aligned} E\left(D_t^h\right) &= \frac{1}{t}\sum_{z=1}^{t}\left(h_{max} - c_h z\right) \\ &= h_{max} - \frac{1}{t}c_h\frac{t(t+1)}{2} \\ &= h_{max} - \frac{t+1}{2}c_h\ . \end{aligned} \tag{A.1}$$

It is easy to see the variance based on the expected hoisting distance.

$$\begin{aligned} V\left(D_t^h\right) &= \frac{1}{t}\sum_{z=1}^{t}\left(h_{max} - c_h z\right)^2 - E\left(D_t^h\right)^2 \\ &= h_{max}{}^2 - h_{max}c_h\left(t+1\right) + c_h{}^2\frac{1}{6}\left(t+1\right)\left(2t+1\right) - \left(h_{max} - \frac{t+1}{2}c_h\right)^2 \\ &= \frac{t^2-1}{12}c_h{}^2\ . \end{aligned} \tag{A.2}$$

APPENDIX B. ESTIMATION OF THE VARIANCE OF CYCLE TIME OF THE RECEIVING OPERATION

The distance between any two random points in a block can be expressed by

$$Z = \left|X - Y\right|. \tag{B.1}$$

Its cumulative distribution function and probability density function can be derived as follows:

$$F\left(Z\right) = \frac{b_l{}^2 - \left(b_l - z\right)^2}{b_l{}^2},\ \text{and} \tag{B.2}$$

$$f\left(z\right) = \frac{2\left(b_l - z\right)}{b_l{}^2}\ . \tag{B.3}$$

Thus, we can have the expectation and the variance of the distance between two positions in a block as follows:

$$E(Z) = \frac{1}{3} b_l \text{, and} \tag{B.4}$$

$$V(Z) = \frac{1}{18} b_l^2 . \tag{B.5}$$

The distance between the TP position at the side of a bay and a random stacking position in the same bay can be expressed by a uniform distribution as follows:

$$Z \sim U(d_c, d_c + b_w). \tag{B.6}$$

By using statistical theory, we can see the expectation and the variance of the distance as follows:

$$E(Z) = \frac{1}{2} b_w \text{, and} \tag{B.7}$$

$$V(Z) = \frac{1}{12} b_w^2 . \tag{B.8}$$

Thus, the variance of cycle time for the receiving operation can be evaluated by using equation (B.4), (B.5), (B.7), and (B.8) as follows:

$V(T_R) = V(T_g) + V(T_t) + V(T_h)$. Thus,

$$V(T_R) = \frac{b_l^2}{18}\left(\frac{1}{v_g^e}\right)^2 + \frac{b_w^2}{12}\left(\left(\frac{1}{v_t^e}\right)^2 + \left(\frac{1}{v_t^l}\right)^2\right) + V(D_t^h)\left(\left(\frac{1}{v_h^e}\right)^2 + \left(\frac{1}{v_h^l}\right)^2\right). \tag{10}$$

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